Key

Math 4 Name ______________

5-9 Quotient Rule Date_____

Goal: I can use the quotient rule to find derivatives of functions.

Today you'll learn another rule that will help you find the derivative for functions of a different form. Take a look at the examples below and see if you can fill in the blanks for the quotient rule. Just like the last lesson, only the first step of work is shown in these two examples. We expect you to completely simplify your answers.

Example 1: $f(x)=\dfrac{x^2}{3x-1}$

$$f'(x)=\frac{(3x-1)\cdot 2x-x^2\cdot 3}{(3x-1)^2}$$

Example 2: $f(x)=\dfrac{7x+5}{3x^4-6}$

$$f'(x)=\frac{(3x^4-6)\cdot 7-(7x+5)\cdot(12x^3)}{(3x^4-6)^2}$$

Quotient Rule: If $f(x)=\dfrac{g(x)}{h(x)}$, then $f'(x)=\dfrac{\overset{lo}{\boxed{h(x)}}\cdot\overset{di\ hi}{\boxed{g'(x)}}-\overset{hi}{\boxed{g(x)}}\cdot\overset{di\ lo}{\boxed{h'(x)}}}{\underset{lo\ lo}{\boxed{h^2(x)}}}$

$h^2(x)=[h(x)]^2$

Example 3: $f(x)=\dfrac{3x^2+5}{7x-4}$

$$f'(x)=\frac{(7x-4)(6x)-(3x^2+5)(7)}{(7x-4)^2}=\frac{42x^2-24x-21x^2-35}{(7x-4)^2}$$

$$=\boxed{\frac{21x^2-24x-35}{(7x-4)^2}}$$

In class practice: For each function, find the first derivative.

1. $f(x)=\dfrac{(2x^2+3x-5)}{x^3}$

$$f'(x)=\frac{x^3(4x+3)-(2x^2+3x-5)(3x^2)}{(x^3)^2}$$

$$=\frac{4x^4+3x^3-6x^4-9x^3+15x^2}{x^6}$$

$$=\frac{-2x^4-6x^3+15x^2}{x^6}=\boxed{\frac{-2x^3-6x^2+15x}{x^5}}$$

2. $f(x)=\dfrac{5x+6}{12}$

$$f'(x)=\frac{12(5)-(5x+6)\cdot 0}{12^2}$$

$$=\frac{60-0}{144}$$

$$=\boxed{\frac{5}{12}}$$

3. $f(x)=\dfrac{7x+4}{2x^2-5}$

$$f'(x)=\frac{(2x^2-5)(7)-(7x+4)(4x)}{(2x^2-5)^2}$$

$$=\frac{14x^2-35-28x^2+16x}{(2x^2-5)^2}$$

$$=\boxed{\frac{-14x^2+16x-35}{(2x^2-5)^2}}$$

4. $y=\dfrac{4x+7}{4x+7}$

$$\frac{dy}{dx}=\frac{(4x+7)(4)-(4x+7)(4)}{(4x+7)^2}$$

$$=\frac{0}{(4x+7)^2}$$

$$=\boxed{0}$$

More practice! Find the first derivative for each function. Show all work!

1. $f(x)=\dfrac{x^2}{(2x+5)}$

$$f'(x)=\frac{(2x+5)(2x)-(x^2)(2)}{(2x+5)^2}$$

$$=\frac{4x^2+10x-2x^2}{(2x+5)^2}$$

$$=\boxed{\frac{2x^2+10x}{(2x+5)^2}}$$

2. $f(x)=\dfrac{3x-1}{x^2}$

$$f'(x)=\frac{x^2\cdot 3-(3x-1)(2x)}{x^4}$$

$$=\frac{3x^2-6x^2+2x}{x^4}$$

$$=\frac{-3x^2+2x}{x^4}$$

$$=\boxed{\frac{-3x+2}{x^3}}$$

3. $f(x)=\dfrac{2x^7-3x^3+24}{2x^7-3x^3+24}$

$$f(x)=1$$

$$\boxed{f'(x)=0}$$

4. $y=\dfrac{x^2}{x-1}$

$$\frac{dy}{dx}=\frac{(x-1)\cdot 2x-x^2(1)}{(x-1)^2}$$

$$=\frac{2x^2-2x-x^2}{(x-1)^2}$$

$$=\boxed{\frac{x^2-2x}{(x-1)^2}}$$

5. $y=\dfrac{2x-1}{x^2+1}$

$$\frac{dy}{dx}=\frac{(x^2+1)(2)-(2x-1)(2x)}{(x^2+1)^2}$$

$$=\frac{2x^2+2-4x^2+2x}{(x^2+1)^2}$$

$$=\boxed{\frac{-2x^2+2x+2}{(x^2+1)^2}}$$

6. $y=\dfrac{x-1}{x^2+1}$

$$\frac{dy}{dx}=\frac{(x^2+1)(1)-(x-1)(2x)}{(x^2+1)^2}$$

$$=\frac{x^2+1-2x^2+2x}{(x^2+1)^2}$$

$$=\boxed{\frac{-x^2+2x+1}{(x^2+1)^2}}$$

7. $y=\dfrac{(3x^2-2x+1)}{(2x-1)}$

$$\frac{dy}{dx}=\frac{(2x-1)(4x-2)-(3x^2-2x+1)(2)}{(2x-1)^2}$$

$$=\frac{8x^2-4x\cancel{-4x}\cancel{+2}-6x^2\cancel{+4x}\cancel{-2}}{(2x-1)^2}$$

$$=\boxed{\frac{2x^2-4x}{(2x-1)^2}}$$

8. $y=\dfrac{x+1}{x-1}$

$$\frac{dy}{dx}=\frac{(x-1)(1)-(x+1)(1)}{(x-1)^2}$$

$$=\frac{x-1-x-1}{(x-1)^2}$$

$$=\boxed{\frac{-2}{(x-1)^2}}$$